Syllabus and Call for Registration: Complex Geometry

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Call for Registration

Registration is currently open for this intensive summer course in Complex Geometry. The curriculum is designed for doctoral students and researchers, proceeding at an accelerated pace with two lectures per day to conclude the syllabus efficiently.

Prospective participants must register prior to the commencement of the course. To formally register, please contact the departmental administration directly.

Course Information

- **Field of Higher Education:** 4. Natural Sciences, Mathematics, and Informatics
- **Professional Field:** 4.5. Mathematics
- **Doctoral Program:** Geometry and Topology
- **Location:** Sofia
- **Total Hours:** 30 hours of lectures (15 lectures, 2 hours each)
- **Credits:** 20

Prerequisites

Real Analysis, Complex Analysis of one variable, and Differential Geometry.

Course Objectives and Competences

The primary objective of the course is to establish the foundations of complex geometry from differential-geometric and analytic perspectives, utilizing the language of sheaves and their cohomology. Participants will acquire a rigorous understanding of Hermitian and Kähler manifolds, the fundamentals of Hodge theory, and the application of these concepts to standard manifolds and their Hodge structures.

Thematic Content and Lecture Schedule

The course spans 15 lectures, with classes held twice daily (10:00 – 12:00 and 14:00 – 16:00) to ensure a focused and continuous exposition. The schedule strictly observes planned institutional recesses.

Lecture 1: **[July 16, 10:00 – 12:00] Fundamentals of Several Complex Variables.** Holomorphic functions of several variables. Osgood's lemma. Multivariate Cauchy integral formula. Morera's theorem and Liouville's theorem.

Lecture 2: **[July 16, 14:00 – 16:00] Extension Phenomena.** Local properties of holomorphic functions. Riemann's extension theorem for removable singularities across analytic sets. Hartogs's extension theorem for $n \geq 2$.

Lecture 3: **[July 17, 10:00 – 12:00] Local Theory and Analytic Sets.** Properties of the ring of germs of holomorphic functions. Weierstrass Preparation and Division theorems. Definition and local structure of analytic sets.

Lecture 4: **[July 17, 14:00 – 16:00] Complex Manifolds.** Concept of a complex manifold. Submanifolds and holomorphic vector bundles. The blow-up of a point. Standard examples including projective spaces and Grassmannians.

Lecture 5: **[July 27, 10:00 – 12:00] Differential Forms.** Calculus on complex manifolds. Holomorphic (p, q)-forms. The ∂ and $\bar{\partial}$ operators. Holomorphic Poincaré lemma.

Lecture 6: **[July 27, 14:00 – 16:00] Hermitian Geometry.** Hermitian manifolds. The Fubini-Study metric on complex projective space. Wirtinger's theorem and volumes of complex submanifolds.

Lecture 7: **[July 28, 10:00 – 12:00] Sheaves and Cohomology.** Introduction to sheaf theory. Čech cohomology. Resolutions of sheaves. Fine and soft sheaves.

Lecture 8: **[July 28, 14:00 – 16:00] Dolbeault Cohomology.** The Dolbeault resolution. Dolbeault's theorem establishing the isomorphism between sheaf cohomology and Dolbeault cohomology. Applications.

Lecture 9: **[July 29, 10:00 – 12:00] Harmonic Theory on Manifolds.** The Hodge star operator. The Laplace operator on a Riemannian manifold. Existence of harmonic representatives for cohomology classes.

Lecture 10: **[July 29, 14:00 – 16:00] Global Embedding Theorems.** Chow's theorem on closed analytic subvarieties of projective space. The Kodaira embedding theorem characterizing projective algebraic manifolds.

Lecture 11: **[July 30, 10:00 – 12:00] Kähler Manifolds.** Definition of Kähler metrics. Complex harmonic theory. The anti-holomorphic Laplace operator. Harmonic representatives of Dolbeault classes.

Lecture 12: **[July 30, 14:00 – 16:00] Kähler Identities.** Characterization of Kähler metrics. Commutator relations among the operators L, Λ, ∂, $\bar{\partial}$, ∂^*, and $\bar{\partial}^*$. Corollaries of the Kähler identities.

Lecture 13: **[August 3, 10:00 – 12:00] Hodge Theory.** The Hodge decomposition theorem. The Hodge diamond and topological restrictions on Kähler manifolds. Examples of Kähler manifolds and their Hodge structures.

Lecture 14: **[August 3, 14:00 – 16:00] The Calabi Conjecture.** Introduction to the complex Monge-Ampère equation. Formulation of the conjecture regarding the existence of Ricci-flat Kähler metrics.

Lecture 15: **[August 4, 10:00 – 12:00] Yau's Theorem and Special Holonomy.** Solution of the complex Monge-Ampère equation. A priori estimates (C^0, C^2, and C^3). Applications to Calabi-Yau manifolds and geometries with special holonomy.

Evaluation Criteria

The final examination spans 4 hours and consists of a written and an oral component.

- **Written Exam:** The student addresses two questions drawn from the syllabus to demonstrate their theoretical comprehension and analytic execution.
- **Oral Exam:** The student addresses questions from the examination committee related to the broader course content.

The final grade is evaluated on a scale from 2 to 6 (with precision up to 0.5), assessing the student's conceptual comprehension, mathematical rigor, and capability to apply the theory.

Recommended Bibliography

1. Schnell, C. *A Graduate Course on Complex Manifolds.* Stony Brook University, 2010. Available at `http://www.math.stonybrook.edu/~cschnell/`
2. Griffiths, P., and Harris, J. *Principles of Algebraic Geometry.* Wiley-Interscience, 1978.
3. Huybrechts, D. *Complex Geometry: An Introduction.* Springer, 2004.
4. Joyce, D. *Compact Manifolds with Special Holonomy.* Oxford University Press, 2000.