

Some properties of the Schouten-van Kampen connection on the tangent bundle with general natural metric

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In this presentation our main tool is the Schouten–van Kampen connection $\bar{\nabla}$ associated to the Levi-Civita connection ∇ of a general natural metric G on the total space TM of the tangent bundle of a Riemannian manifold (M, g) . We determine the mentioned Schouten–van Kampen connection, the conditions under which it is torsion free and the expressions of the general natural metrics for which the Levi-Civita connection coincides with its associated Schouten–van Kampen connection. We provide the characterization of the the general natural α -structures parallel with respect to the torsion free Schouten-van Kampen connection on TM and the characterization of the (para-)Kähler manifolds (TM, G, A) whose general natural α -structure A is parallel with respect to both connections ∇ and $\bar{\nabla}$.

Then, we investigate the manifolds $(TM, \bar{\nabla}, G)$ that are quasi-statistical manifolds (also called statistical manifolds with torsion), for which we show that the base manifold must be a space form when G is a proper general natural metric and locally flat when G is a natural diagonal metric. We prove that $(TM, \bar{\nabla}, G)$ is a statistical manifold (without torsion) if and only if the base manifold is locally flat and the metric G is a proper general natural metric with two possible expressions or a natural diagonal metric depending on two arbitrary nonzero smooth real functions of the energy density t and on an arbitrary nonzero real constant. Moreover, we show that there exists one family of proper general natural metrics such that $(TM \setminus \{0\}, \bar{\nabla}, G)$ is a quasi-statistical manifold. A necessary condition for the existence of such a manifold is that the base manifold has negative constant sectional curvature.