

# Universal Bounds on Energy and Polarization of Spherical Codes and Designs

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(Joint work with Peter Boyvalenkov, Peter Dragnev, Douglas Hardin and Edward Saff)

We present results on universal (in sense of Levenshtein [7] and Cohn-Kumar [4]) bounds on potential energy and polarization of spherical codes and designs [1]. A framework based on the Delsarte-Yudin linear programming approach [5, 8] for improving some universal lower bounds for the minimum energy of spherical codes of prescribed dimension and cardinality is introduced [2]. In particular, the results extend the Levenshtein framework [7, 6]. Next we investigate the  $N$ -point min-max and the max-min polarization problems on the sphere. We derive universal lower and upper bounds on the polarization of spherical codes and designs of fixed dimension, strength, and cardinality [3]. We show that the potentials of most of the known sharp codes attain the universal lower bounds for polarization for spherical  $\tau$ -designs.

## References

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<sup>1</sup>The research was supported, in part, by the European Union-NextGenerationEU, through the National Recovery and Resilience Plan of the Republic of Bulgaria, project SUMMIT BG-RRP-2.004-0008-C01.