Women in Mathematics in South-Eastern Europe
December 18 – 20, 2023, Sofia, Bulgaria

On the $\mathrm{GL}(n)$-module Structure of Several Classes of Relatively Free Algebras

Elitza Hristova
Institute of Mathematics and Informatics,
Bulgarian Academy of Sciences

Let $K\langle X_n\rangle$ denote the free associative algebra generated by a set X_n with n elements over a field K of characteristic 0. A T-ideal I in $K\langle X_n\rangle$ is any ideal which is closed under all K-algebra endomorphisms of $K\langle X_n\rangle$. The quotient $K\langle X_n\rangle/I$ is called a relatively free algebra of rank n. There is a natural action of the group $\mathrm{GL}(n,K)$ on the relatively free algebra $K\langle X_n\rangle/I$ and one question in the theory of PI-algebras is to determine the $\mathrm{GL}(n,K)$-module structure of $K\langle X_n\rangle/I$ for different classes of T-ideals I. In this talk, we consider first the case $I = I_{p+1}$, where for any positive integer p, I_{p+1} denotes the ideal in $K\langle X_n\rangle$ generated by all commutators of length $p+1$. The $\mathrm{GL}(n,K)$-module structure of $K\langle X_n\rangle/I_{p+1}$ is known for $p = 1, 2, 3, 4$. In the talk, we discuss some results on the $\mathrm{GL}(n,K)$-module structure of $K\langle X_n\rangle/I_{p+1}$ for arbitrary values of p. In particular, we give a bound on the values of partitions λ such that the irreducible $\mathrm{GL}(n,K)$-module with highest weight λ appears with nonzero multiplicity in the decomposition of $K\langle X_n\rangle/I_{p+1}$ as a $\mathrm{GL}(n,K)$-module. Then we extend these results to the case when I is a product of T-ideals of the form I_{p+1}. We discuss also applications related to the algebras of G-invariants in $K\langle X_n\rangle/I_{p+1}$, where we take $K = \mathbb{C}$ and G to be one of the classical complex groups $\mathrm{SL}(n,\mathbb{C})$, $\mathrm{O}(n,\mathbb{C})$, $\mathrm{SO}(n,\mathbb{C})$, or $\mathrm{Sp}(2k,\mathbb{C})$ (for $n = 2k$).