

Almost Complex and Hypercomplex Norden Structures on the Cotangent Bundle

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The Riemann extension, introduced by E. K. Patterson and A. G. Walker (1952), is a semi-Riemannian metric with a neutral signature on the cotangent bundle T^*M of a smooth manifold M , induced by a symmetric linear connection ∇ on M . In this paper we deal with a natural Riemann extension \bar{g} which is a generalization (due to M. Sekizawa and O. Kowalski) of the Riemann extension. We construct an almost complex structure \bar{J} on the cotangent bundle T^*M of an almost complex manifold (M, J, ∇) with a symmetric linear connection ∇ such that (T^*M, \bar{J}, \bar{g}) is an almost complex manifold, where the natural Riemann extension \bar{g} is Norden metric. We obtain necessary and sufficient conditions for (T^*M, \bar{J}, \bar{g}) to belong to the main classes of the Ganchev-Borisov classification of the almost complex manifolds with Norden metric. We also examine the cases when the base manifold is an almost complex manifold with Norden metric or it is a complex manifold (M, J, ∇') endowed with an almost complex connection ∇' ($\nabla'J = 0$). We investigate the harmonicity with respect to \bar{g} of the almost complex structure \bar{J} , according to type of the base manifold. Moreover, we define an almost hypercomplex structure $(\bar{J}_1, \bar{J}_2, \bar{J}_3)$ on the cotangent bundle T^*M^{4n} of an almost hypercomplex manifold $(M^{4n}, J_1, J_2, J_3, \nabla)$ with a symmetric linear connection ∇ . The natural Riemann extension \bar{g} is Hermitian metric with respect to \bar{J}_1 and Norden metric with respect to \bar{J}_2 and \bar{J}_3 .