

# Workshop “New mathematical methods in solvable models and gauge/string dualities” 14-20 August 2022

## TITLES AND ABSTRACTS OF THE TALKS

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**Hrachya Babujian (Zoom)**  
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*Asymptotic Factorization of the  $n$ -particle  $SU(N)$  form factors.  
(Talk impossible because of technical issues)*

**Abstract:** We investigate the high energy behavior of the  $SU(N)$  chiral Gross-Neveu model in  $1 + 1$  dimensions. The model is integrable and matrix elements of several local operators (form factors) are known exactly. The form factors show rapidity space clustering, which means factorization, if a group of rapidities is shifted to infinity. We analyze this phenomenon for the  $SU(N)$  model. For several operators the factorization formulas are presented explicitly.

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**Zoltan Bajnok**  
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*CFT 3pt-functions from integrability*

**Abstract:** In my talk I will explain how the diagonal 3pt-functions in  $1+1$  dimensional conformal field theories might be expressed in terms of the integrable structure of the underlying CFT. The idea is to introduce a massive integrable perturbation, calculate the spectrum and expectation values, and then investigate their CFT limit. In doing so, we expand the ground-state energy at small volume, a problem which resisted solutions for decades. As a result, we can express 3-point functions and mass-coupling relations purely in terms of the CFT kink functions.

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**Eldad Bettelheim**  
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*Exact Matrix Elements of the Field Operator in the Thermodynamic Limit of the Lieb-Liniger Model*

**Abstract:** We study a matrix element of the field operator in the Lieb-Liniger model using the Bethe ansatz technique coupled with a functional approach to compute Slavnov determinants. We obtain the matrix element exactly in the thermodynamic limit for any coupling constant  $c$ , and compare our results to known semiclassics at the limit  $c \rightarrow 0$ .

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**Nikolai Bobev**

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*Large N Partition Functions, Holography, and Black Holes*

**Abstract:** I will discuss the large N behavior of partition functions of the ABJM theory on compact Euclidean manifolds. I will pay particular attention to the  $S^3$  free energy and the topologically twisted index for which I will present closed form expressions valid to all order in the large N expansion. These results have important implications for holography and the microscopic entropy counting of AdS<sub>4</sub> black holes which I will discuss. I will also briefly discuss generalizations to other SCFTs arising from M2-branes.

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**Joao Caetano**

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*Unoriented QFTs*

**Abstract:** In two dimensions, one can study QFTs on unoriented manifolds by introducing crosscaps. This defines a class of states called crosscap states which share similarities with boundary states. In this talk, I will show that integrable theories remain integrable in the presence of crosscaps, and this allows to exactly determine the crosscap state. In four dimensions, the analog is to place QFTs on the real projective space, the simplest unoriented manifold. I will show how to do this in the example of N=4 Supersymmetric Yang-Mills, discuss its holographic description and present a new solvable setup of AdS/CFT.

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**Philippe Di Francesco**

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*Triangular ice: combinatorics and limit shapes*

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**Simon Ekhammar**

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*Q-systems, QQ-relations and T-functions beyond  $gl[N]$*

**Abstract:** The success and utility of the Quantum Spectral Curve in N=4 SYM raises the question: can we extend it to other theories? This task requires generalising the Q-system that underlies the Quantum Spectral Curve to symmetries different from  $gl[N]$ . I will talk about recent and ongoing work with Dmytro Volin exploring these Q-systems. In general, the symmetry of the Q-system no longer coincides with that of the integrable model but it still encodes non-trivial QQ-relations leading to standard Bethe equations. Furthermore, the Q-system naturally gives compact expressions for T-functions solving Hirota equations and useful quantization conditions. As an application I will discuss, for simple examples, how to impose appropriate analytic properties and find efficient ways to solve for the spectrum of (twisted) rational spin

chains.

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**Gwenaël Ferrando**  
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*Large-Twist Limit for Any Operator in N=4 SYM*

**Abstract:** The fishnet theory was obtained as a strongly twisted, weakly coupled limit of N=4 SYM. Though still non-trivial, it is much simpler than the original N=4 SYM theory. The appearance of integrability is better understood (at least for the spectrum), and the holographic dual was constructed from first principles. Both can be tied to the existence of an iterative structure for some of the correlators. However, the fishnet theory only contains two scalar fields, and most of the operators of the original theory are now protected. In particular, the gauge boson has completely decoupled. We argue that it is possible to devise a double-scaling limit for any operator in N=4 SYM. We consider several examples that were protected in the fishnet theory, including operators containing the gauge boson. We show that the generic situation involves some type of mixing with other operators sharing the same R-symmetry charges. The graph-building operator is promoted to an infinite matrix. When focusing on a particular double-scaling limit, only a finite submatrix is relevant. This work is a first step towards a systematic expansion of N=4 SYM around the large-twist limit.

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**Fridrik Freyr Gautason**  
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*Wilson loops in 5D SYM and Holography*

**Abstract:** I will discuss Maximally supersymmetric Yang-Mills theory in five dimensions. Using supersymmetric localization, the expectation value of a BPS Wilson loop operator can be computed exactly at large rank and strong coupling. In the talk I will review this result and explain how to reproduce it in string theory. This constitutes a precision test of holography for a non-conformal theory beyond leading order.

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**Alessandro Georgoudis**  
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*Hexagons for short operators: A hands on approach*

**Abstract:** In this talk I will present a recent integrability-based conjecture for the three-point functions of single-trace operators in planar N=4 super-Yang-Mills theory at finite coupling, in the case where two operators are protected. This proposal is based on the hexagon representation for structure constants of long operators, which we complete to incorporate operators of any length using data from the TBA/QSC formalism. I will then present how to perform perturbative computations, showcase

some results and discuss further possible applications.

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**Tamas Gombor**

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*Wrapping corrections for long range spin chains*

**Abstract:** In this talk, I show a construction of transfer matrices for long range spin chains. These transfer matrices define a large set of conserved charges for every length of the spin chain. These charges agree with the original definition of long range spin chains for infinite length. However, this new construction works for every length, providing the definition of integrable finite size long range spin chains. The properties of these finite size Hamiltonians are similar as expected from the wrapping corrections of the planar  $N=4$  super Yang-Mills.

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**Nikolay Gromov**

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Bootstrability in Defect CFT: Integrated Correlators and Sharper Bounds

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**Kiril Hristov**

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Gravitational building blocks and ABJM at finite  $N$

**Abstract:** We propose a general fixed-point formula for the on-shell action of supersymmetric backgrounds in terms of gravitational building blocks. The building block of 4d  $N=2$  supergravity takes the form of a Nekrasov-like partition function with equivariant parameters related to the higher-derivative expansion of the prepotential. We apply this proposal to the effective supergravity description of M-theory on  $S^7/Z_k$ . Utilizing known localization results for the holographically dual ABJM theory, we are able to determine the complete infinite derivative expression of the  $AdS_4$  supergravity prepotential. We thus find holographic predictions for a number of ABJM observables.

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**Rinat Kedem**

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*Duality and integrability of the quantum  $Q$ -systems for classical root systems*

**Abstract:** Quantum  $Q$ -systems are discrete integrable systems obtained as quantizations of classical relations among characters of quantum affine algebra representations. They are also mutation relations for cluster variables in certain quantum cluster algebras. These systems have many remarkable properties, and here, I will explain the relation to the  $q$ -Whittaker limit of Macdonald operators and

spherical DAHAs, and quantum relativistic Toda systems. This is joint work with Philippe Di Francesco.

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**Nat Levine**

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*Universal 1-loop beta functions for integrable sigma-models*

**Abstract:** I will present a simple, new method for the 1-loop renormalization of integrable sigma-models. By treating equations of motion and Bianchi identities on an equal footing, we will derive universal formulae for the beta functions in terms of the Lax connection, generalizing case-by-case computations in the literature. We will discuss potential applications to (i) proving that integrable models are renormalizable and (ii) finding supergravity solutions using 2d integrability. Based on work to appear.

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**Fedor Levkovich-Maslyuk**

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*Critical phenomena for dually weighted graphs and spanning forests via matrix models*

**Abstract:** I will discuss two nontrivial problems of 2d quantum gravity / planar graph counting that are solvable by reduction to matrix models. The first one is summing over planar graphs with weights for both vertices and faces. In this setting one can control the geometry efficiently and obtain in the continuum limit a nontrivial 2d theory of quantum gravity ( $R^2$  type). I will compute the disk partition function in this model and show that it interpolates between pure gravity and the regime of almost flat surfaces with a gas of conical defects.

The second problem is counting rooted spanning forests on planar trivalent graphs. Equivalently this corresponds to massive spinless fermions interacting with 2d quantum gravity. I will show how this model can be solved, demonstrate that it interpolates between  $c=-2$  and  $c=0$  regimes, and explore its phase structure and critical behavior.

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**Andrii Liashyk**

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**George Linardopoulos**

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*String integrability of defect CFT and dynamical reflection matrices*

**Abstract:** The D3-D5 and D2-D4 probe-brane systems with nonzero worldvolume flux are holographically dual to  $N=4$  super Yang-Mills and ABJM theory in the presence of half-BPS domain walls. The two domain wall systems are thought to be integrable; the evidence comes mainly from the study of correlation functions at

weak coupling. In the present talk we show that the string theory duals of these systems are classically integrable. In other words, the string boundary conditions on the probe branes preserve the integrability of the corresponding Green-Schwarz sigma models. This finding suggests that the dual domain wall systems are integrable to all loop orders and for any value of the bond dimension.

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**Andrei Marshakov**

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*Supersymmetric gauge theories, quivers and Painleve equations*

**Abstract:** We consider a relation between supersymmetric gauge theories in 4(+1)d and integrable systems on cluster varieties, defined by mutation classes of the Poisson quivers. We conjecture, that these systems generally belong to the class of cluster reductions, and demonstrate, in particular, how it allows to fill the gap in the known gauge/Painleve correspondence.

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**Enrico Olivucci**

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*Integrability of correlators in N=4 SYM: null limit*

Abstract: I will review recent results about the exact solution of multi-point correlators of light 1/2-BPS operators in N=4 SYM theory via a system of coupled Toda field theory equations.

I will speculate on the validity of these results beyond N=4 SYM and beyond the null limit, towards general kinematics.

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**Balazs Pozsgay**

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**Radoslav Rashkov**

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**Paul Ryan**

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*Correlation functions and functional separation of variables*

**Abstract:** In recent years it has become apparent that correlation functions in N=4 SYM and its cousins take a drastically simplified form when expressed in terms of the quantum spectral curve Q-functions. This is similar to the situation in integrable spin chains using Sklyanin's separated variables and has been one of the main driving factors in the development of the separation of variables (SoV) program to high-rank integrable systems.

The old operator-based SoV techniques of Sklyanin have recently been supplemented

with a "functional" SoV (FSoV) which has proven its power in both spin chains and field theory by allowing to easily compute scalar products and certain diagonal form-factors. In this talk I will describe how to use FSoV to extend beyond the diagonal case in  $SL(N)$  spin chains. This is done by introducing a special family of operators, which we call "principal operators". These are a subset of Yangian generators, and we show that they generate the complete family of observables of the model. By combining the FSoV with a novel technique which we call "character projection" we are able to compute all matrix elements of these operators. The matrix elements take the form of highly compact determinants in Q-functions which contain all state-dependent information unlike in the various Bethe-ansatz based approaches. The approach is highly general, and works for both finite and infinite-dimensional representations alike.

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**Fedor Smirnov**

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*Fermionic basis and massless flows*

**Abstract:** The talk is based on unpublished work done in collaboration with Stefano Negro. The fermionic basis provides a description of the space of local operators for integrable models. In the conformal case the elements of the fermionic basis can be identified with the Virasoro descendants. We consider the massless flows between two CFT, and show how the fermionic basis allow to identify the IR image of a given UV local operator.

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**Konstantin Zarembo**

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*Introduction to AdS/CFT integrability*

I will review AdS/CFT integrability from the beginning onwards.  
Hopefully ending with recent developments on correlation functions.

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**Deliang Zhong (Weizmann)**

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*Line Operators in Chern-Simons-Matter Theories and Bosonization in Three Dimensions*

**Abstract:** We study Chern-Simons theories at large  $N$  with either bosonic or fermionic matter in the fundamental representation. The most fundamental operators in these theories are mesonic line operators, the simplest example being Wilson lines ending on fundamentals. We classify the conformal line operators along an arbitrary smooth path as well as the spectrum of conformal dimensions and transverse spins of their boundary operators at finite 't Hooft coupling. These line operators are shown to satisfy first-order chiral evolution equations, in which a smooth variation of the path is given by a factorized product of two line operators. We argue that this equation

together with the spectrum of boundary operators are sufficient to uniquely determine the expectation values of these operators. We demonstrate this by bootstrapping the two-point function of the displacement operator on a straight line. We show that the line operators in the theory of bosons and the theory of fermions satisfy the same evolution equation and have the same spectrum of boundary operators.